A Novel Approach for the Characterization of FSK Low Probability of Intercept Radar Signals Via Application of the Reassignment Method

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***Abstract*—A novel approach of employing the reassignment method as a signal analysis technique for use in digital intercept receivers for the analysis of frequency shift keying (FSK) low probability of intercept (LPI) radar signals is taken. Digital intercept receivers are currently moving away from Fourier analysis techniques using the FFT as the basic tool for detecting and extracting parameters of LPI radar signals, and are moving towards classical time-frequency analysis techniques (e.g. spectrogram, Wigner-Ville distribution (WVD)). These classical techniques, though an improvement over Fourier analysis techniques, suffer from a lack of readability, due to poor time-frequency localization and cross-term interference. This lack of readability can lead to inaccurate detection and parameter extraction of these signals, potentially threatening survivability of the intercept receiver environment in certain situations. The reassignment method, with its 'squeezing' and 'smoothing' qualities, is considered as an innovative and improved signal analysis technique for this scenario. The FSK signal is used for testing. Experimental modeling and simulation results are subjectively assessed and measured via seven different metrics (including time-frequency localization, a novel metric). Reassignment method results are compared to classical time-frequency analysis results. The results confirm the improved readability of the reassignment method, and consequently, more accurate signal detection and parameter extraction metrics. These results have the potential of improving timeliness of signal characterization, and as a result, increasing aircraft survivability in particular situations.**

***Keywords*—reassignment method; low probability of intercept; readability; time-frequency analysis; digital intercept receivers.**

I. INTRODUCTION

Fourier analysis techniques using the FFT have been employed as the basic tool of the digital intercept receiver for detecting and extracting parameters of low probability of intercept (LPI) radar signals, and make up a majority of the digital intercept receiver techniques that are currently in the fleet [1]. When a practical non-stationary signal (such as an LPI radar signal) is processed, the Fourier Transform cannot efficiently analyze and process the time-varying characteristics of the signal's frequency spectrum, because time and frequency information cannot be combined to tell how frequency content is changing in time. The non-stationary nature of the received radar signal mandates the use of some form of time-frequency (TF) analysis for signal detection and parameter extraction.

Two of the more popular classical TF analysis techniques are the Wigner-Ville distribution (WVD) and the spectrogram. The WVD exhibits the highest signal energy concentration [2], but has the worse cross-term interference, which can severely limit the readability of a TF representation. The spectrogram has poorer time-frequency localization but less cross-term interference than the WVD, and its cross-terms are limited to regions where the signals overlap [3]. Currently, for digital intercept receivers, these classical TF analysis techniques are primarily at the lab phase [1].

Though classical TF analysis techniques are an improvement over Fourier analysis techniques, they suffer in general from poor TF localization and cross-term interference. This may result in degraded readability of TF representations, potentially leading to inaccurate LPI radar signal detection and parameter extraction metrics. This, in turn, can place the intercept receiver environment in harm's way.

A promising avenue for addressing these difficulties of the classical TF analysis techniques is the utilization of the reassignment method – a non-linear, post-processing signal processing technique which can improve the localization of a TF transform (and consequently its readability) by moving its value according to a suitable vector field.

II. THE REASSIGNMENT METHOD

In deriving the reassigned spectrogram, we note that the spectrogram (the magnitude squared of the short-time Fourier transform (STFT – a transform used to determine the frequency of local sections of a signal as it changes over time)) can be defined as a two-dimensional convolution of the WVD of the signal by the WVD of the analysis window, and is given by

$$S_x(t,f;h) = \iint_{-\infty}^{+\infty} W_x(s,\xi) W_h(t-s, f-\xi)\, ds\, d\xi \qquad (1)$$

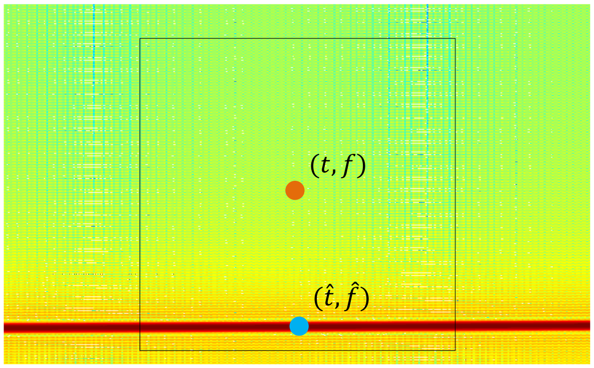

Fig. 1. The reassignment method moves each value of the spectrogram computed at any point (t,f) to another point $(\hat{t},\hat{f})$ which is the center of gravity of the signal energy distribution around (t,f).

(where x is the signal, t is time, f is frequency, h is the window function, and s and ξ are dummy variables delimiting the time-frequency domain about (t,f)). Therefore, the spectrogram reduces the interference terms of the signal's WVD, but at the expense of time and frequency localization. However, a closer look at (1) shows that $W_h(t-s,f-\xi)$ delimits a time-frequency domain at the vicinity of the (t,f) point, inside which a weighted average of the signal's WVD values is performed. The key point of the reassignment principle is that these values have no reason to be symmetrically distributed around (t,f), which is the geometrical center of this domain. Therefore, their average should not be assigned at this point, but rather at the center of gravity of this domain, which is much more representative of the local energy distribution of the signal [4]. Reasoning with a mechanical analogy, the local energy distribution $W_h(t-s,f-\xi)W_x(s,\xi)$ (as a function of s and ξ) can be considered as a mass distribution, and it is much more accurate to assign the total mass (i.e. the spectrogram value) to the center of gravity of the domain rather than to its geometrical center. Another way to look at it is this: the total mass of an object is assigned to its geometrical center, an arbitrary point which except in the very specific case of a homogeneous distribution, has no reason to suit the actual distribution. A much more meaningful choice is to assign the total mass of an object, as well as the spectrogram value, to the center of gravity of their respective distribution [5].

This is exactly how the reassignment method proceeds: it moves each value of the spectrogram computed at any point (t,f) to another point $(\hat{t},\hat{f})$, whose value is given by

$$\hat{t}(x;t,f) = \frac{\iint_{-\infty}^{+\infty} s\, W_h(t-s,f-\xi)W_x(s,\xi)ds\, d\xi}{\iint_{-\infty}^{+\infty} W_h(t-s,f-\xi)W_x(s,\xi)ds\, d\xi} \tag{2}$$

$$\hat{f}(x;t,f) = \frac{\iint_{-\infty}^{+\infty} \xi\, W_h(t-s,f-\xi)W_x(s,\xi)ds\, d\xi}{\iint_{-\infty}^{+\infty} W_h(t-s,f-\xi)W_x(s,\xi)ds\, d\xi} \tag{3}$$

which is the center of gravity of the signal energy distribution around (t,f) [6]. Fig. 1 gives a visual depiction of this.

This leads to the expression for the reassigned spectrogram

$$S_x^{(r)}(t',f';h) = \iint_{-\infty}^{+\infty} S_x(t,f;h)\delta\big(t'-\hat{t}(x;t,f)\big) \cdot \delta\left(f'-\hat{f}(x;t,f)\right) dt\, df \tag{4}$$

whose value at any point (t',f') is the sum of all the spectrogram values reassigned to this point.

One of the most interesting properties of this new distribution is that it also uses the phase information of the STFT, and not only its squared modulus as in the spectrogram. It uses this information from the phase spectrum to sharpen the amplitude estimates in time and frequency. This can be seen from the following expressions of the reassignment operators

$$\hat{t}(x;t,f) = -\frac{d\Phi_x(t,f;h)}{df} \tag{5}$$

$$\hat{f}(x;t,f) = f + \frac{d\Phi_x(t,f;h)}{dt} \tag{6}$$

where $\Phi_x(t,f;h)$ is the phase of the STFT of x (F_x): $\Phi_x(t,f;h) = \arg(F_x(\mathrm{t, f; h}))$. However, these expressions do not lead to an efficient implementation, and have to be replaced by

$$\hat{t}(x;t,f) = t - \Re\left\{\frac{F_x(t,f;T_{h)}F_x^*(t,f;h)}{\left|F_{x(t,f;h}\right|^2}\right\} \tag{7}$$

$$\hat{f}(x;t,f) = f - \Im\left\{\frac{F_x(t,f;D_{h)}F_x^*(t,f;h)}{\left|F_{x(t,f;h}\right|^2}\right\} \tag{8}$$

where (7) is the local group delay, (8) is the local instantaneous frequency, $T_h(t) = t \times h(t)$, and $D_h(t) = \frac{dh}{dt}(t)$. This leads to an efficient implementation for the reassigned spectrogram without explicitly computing the partial derivatives of phase. The reassigned spectrogram may thus be computed by using three STFTs, each having a different window (the window function h; the same window with a weighted time ramp t*h; the derivative of the window function h with respect to time (dh/dt)). Reassigned

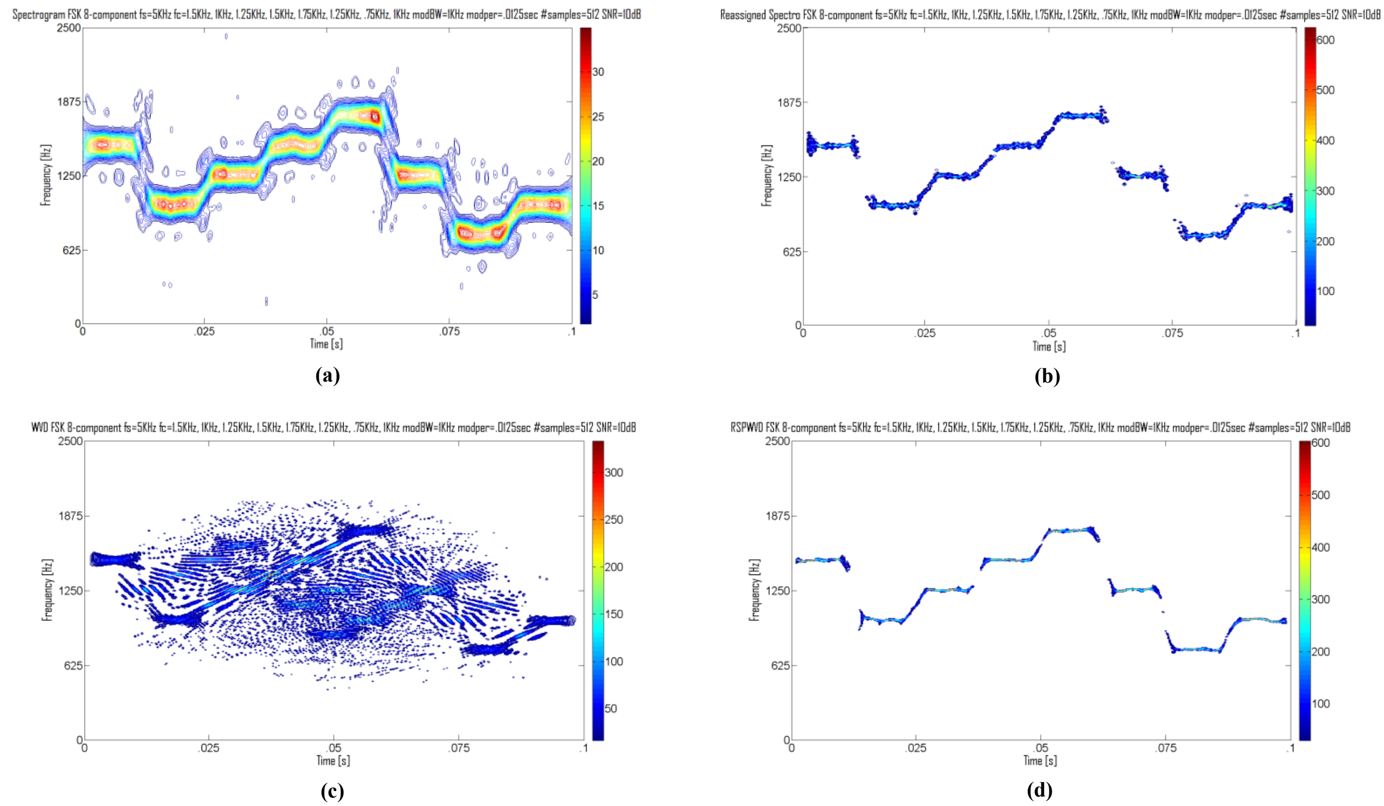

Fig. 2. (a) Spectrogram; (b) reassigned spectrogram; (c) WVD; (d) RSPWVD; each for an 8-component FSK signal with carrier frequencies=1.5KHz, 1KHz, 1.25KHz, 1.5KHz, 1.75KHz, 1.25KHz, 0.75KHz, 1KHz; sampling frequency=5KHz; modulation bandwidth=1KHz; modulation period=12.5ms; number of samples=512; SNR=10dB.

TABLE I.
TEST METRICS COMPARISONS OF FIG. 2 PLOTS – FIRST BETWEEN THE SPECTROGRAM AND THE REASSIGNED SPECTROGRAM, AND THEN BETWEEN THE WVD AND THE RSPWVD (‘*’ DENOTES OUTPERFORMED ITS COUNTERPART; ‘~’ DENOTES PERFORMED ABOUT EQUAL TO ITS COUNTERPART; ‘AVG OF THE 8’ MEANS THE AVERAGE OF THE 8 SIGNAL COMPONENTS OF THE 8-COMPONENT FSK SIGNAL)

Parameters Extracted	**Spectrogram**	**Reassigned Spectrogram**	**WVD**	**RSPWVD**
Carrier Frequency (% error –avg of the 8)	~ 0.11%	~ 0.11%	~ 0.05%	~ 0.07%
Modulation Bandwidth (% error)	24.3%	* 5.74%	6.14%	* 3.5%
Modulation Period (% error –avg of the 8)	~ 10.1%	~ 11.9%	26.95%	* 15.1%
TF Localization (% of y-axis –avg of the 8)	9.3%	* 2.0%	2.3%	* 0.7%
Percent Detection	~ 100%	~ 100%	~ 100%	~ 100%
Lowest Detectable SNR	- 1.0 dB	* - 2.0 dB	- 1.0dB	* - 2.0 dB
Plot (Processing) Time	* 2.0 s	34.0 s	16m:24s	* 22.3 s

spectrograms are therefore very easy to implement, and do not require a drastic increase in computational complexity.

The reassignment principle for the spectrogram allows for a straight-forward extension of its use to other distributions as well [7]. If we consider the general expression of a distribution of the Cohen’s class as a two-dimensional convolution of the WVD, then replacing the particular smoothing kernel $W_h(s,\xi)$ by an arbitrary kernel $\Pi(s,\xi)$ in (1), (2), and (3) gives

$$C_x(t,f;\Pi) = \iint_{-\infty}^{+\infty} \Pi(t-s,f-\xi)W_x(s,\xi)ds\,d\xi \qquad (9)$$

$$\hat{t}(x;t,f) = \frac{\iint_{-\infty}^{+\infty} s\,\Pi(t-s,f-\xi)W_x(s,\xi)ds\,d\xi}{\iint_{-\infty}^{+\infty} \Pi(t-s,f-\xi)W_x(s,\xi)ds\,d\xi} \qquad (10)$$

$$\hat{f}(x;t,f) = \frac{\iint_{-\infty}^{+\infty} \xi\,\Pi(t-s,f-\xi)W_x(s,\xi)ds\,d\xi}{\iint_{-\infty}^{+\infty} \Pi(t-s,f-\xi)W_x(s,\xi)ds\,d\xi} \qquad (11)$$

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leading to the expression for the reassignment of any member of Cohen's class

$$C_x^{(r)}(t',f';\Pi) = \iint_{-\infty}^{+\infty} C_x\,(t,f;\Pi)\delta\big(t' - \hat{t}(x;t,f)\big) \cdot \delta\left(f' - \hat{f}(x;t,f)\right) dt\, df. \quad (12)$$

The resulting reassigned distributions efficiently combine a reduction of the interference terms provided by a well adapted smoothing kernel and an increased concentration of the signal components achieved by the reassignment. In addition, the reassignment operators $\hat{t}(x;t,f)$ and $\hat{f}(x;t,f)$ are almost as easy to compute as for the spectrogram [8].

The reassignment method, which can be applied to most energy distributions, has, in theory, a perfectly localized distribution for chirps, tones, and impulses [5], making it a good candidate for the analysis of certain LPI radar signals, such as the frequency shift keying (FSK) signal, which can be viewed as multiple tones.

The reassignment method can be viewed a helping to build a more readable TF representation. The first step in the process is to reduce (smooth) the cross-term interference. An unfortunate side-effect of this smoothing is that the signal components become 'smeared'. The second step in the process is then to refocus (squeeze) the components which were smeared during the smoothing process. This 'smoothing' and 'squeezing' helps to create a more readable TF representation. Similar work has been accomplished using the reassignment method for a single chirp signal (and in some cases, two parallel chirps) [7]. The FSK signal used in this research presents the extra challenge of cross-term interference, which a single signal does not manifest.

As mentioned, the FSK waveform is analyzed (due to its prevalence as an LPI radar waveform [9]) using the classical TF analysis techniques (WVD, spectrogram) and the reassignment method (reassigned spectrogram, reassigned smoothed pseudo WVD (RSPWVD)). The parameters extracted are: carrier frequencies, modulation bandwidth, modulation period, time-frequency localization (a novel metric, which for an FSK signal, measures the thickness of a signal component (at the center of the component) as a percentage of the entire y-axis), percent detection, lowest detectable SNR and plot (processing) time.

III. FSK LPI WAVEFORM DETECTION AND PARAMETER EXTRACTION

Fig. 2 shows the (a) spectrogram; (b) reassigned spectrogram; (c) WVD; (d) RSPWVD; each for an 8-component FSK signal. Table I shows the test metrics comparisions from these plots (for greater than 600 total test runs), first between the spectrogram and the reassigned spectrogram, and then between the WVD and the RSPWVD.

The top row of Fig. 2 ((a) and (b)) clearly shows the 'squeezing' (localization) quality of the reassignment method, leading to more accurate modulation bandwidth, time-frequency localization (y direction), and lowest detectable SNR metrics for the reassigned spectrogram, as seen in Table I.

The bottom row of Fig. 2 ((c) and (d)) clearly shows the 'smoothing' quality of the reassignment method, which clears up the cross-term interference produced by the WVD, making the TF presentation more readable, and leading to more accurate modulation bandwidth, modulation period, time-frequency localization (y direction), lowest detectable SNR, and plot (processing) time metrics for the RSPWVD, as seen in Table I (note – the WVD is known to be very computationally complex [10], leading to a large plot (processing) time).

IV. CONCLUDING REMARKS

This paper has examined the novel application of the reassignment method as a signal analysis technique for use in digital intercept receivers for the analysis of FSK LPI radar signals. The reassignment method is compared with classical time-frequency analysis techniques (the current state-of-the-art signal analysis techniques for digital intercept receivers) using experimental modeling and simulation results. The results show that the 'smoothing' and 'squeezing' qualities of the reassignment method produce better readability of TF representations than do the classical TF analysis techniques (which suffer from poor TF localization and cross-term interference), which in turn produce more accurate signal detection and parameter extraction metrics. These results have the potential of improving timeliness of signal characterization, and as a result, increasing aircraft survivability in particular situations.

Future work will consist of similar comparative modeling and simulation testing using additional reassignment signal analysis techniques, other types of LPI radar signals, and added classical time-frequency analysis techniques. Also, the algorithm development of the reassignment method will be examined for potential ways to modify/enhance the reassignment method so as to produce even better readability and more accurate metrics than it currently does.

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