

Quantum dynamics of spins coupled by electrons in a one-dimensional channel

 Dmitry Mozyrsky,¹ Alexander Dementsov,² and Vladimir Privman²
¹*Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*
²*Department of Physics, Clarkson University, Potsdam, New York 13699, USA*

(Received 19 September 2005; published 9 December 2005)

We develop a unified theoretical description of the induced interaction and quantum noise in a system of two spins (qubits) coupled via a quasi-one-dimensional electron gas in the Luttinger liquid regime. Our results allow evaluation of the degree of coherence in quantum dynamics driven by the induced indirect exchange interaction of localized magnetic moments due to conduction electrons, in channel geometries recently experimentally studied for qubit control and measurement.

 DOI: [10.1103/PhysRevB.72.233103](https://doi.org/10.1103/PhysRevB.72.233103)

PACS number(s): 85.85.+j, 05.30.-d, 05.60.Gg

Recently, there has been much interest in coherent quantum dynamics of coupled two-level systems (qubits) for quantum information processing. Realizations are sought such that qubit-qubit interactions can be externally controlled over short time scales of quantum “gate functions,” in the parameter regime ensuring that relaxation and decoherence are negligibly small over a large number of gate cycles. There have been several proposals for qubit systems in semiconductor heterostructures, with direct coupling,¹ typically via shared electron wave functions, or indirect coupling, specifically via excitations of the conduction electron gas.² In the latter approaches, the medium that induces the indirect interaction, can also act as a “heat bath” resulting in relaxation and decoherence. Usually, strongly correlated, low-temperature conditions have been assumed³ in order to ensure high degree of coherence. In this work we, for the first time, develop a unified theoretical derivation of the Ruderman-Kittel-Kasuya-Yosida (RKKY) type⁴ induced interaction incorporating the description of relaxation effects resulting from the electron gas “bath.”

A recent experiment⁵ on coupled quantum dots has demonstrated the realizability of indirect interaction for control of two-qubit dynamics. Several experimental setups⁶ suggest that a quasi-one-dimensional (1D) channel geometry for the conduction electron gas is promising for quantum measurement required for quantum computing. Furthermore, there is experimental evidence⁷ of Luttinger liquid behavior in electron transport in quasi-1D structures. Therefore, we are going to consider the 1D-channel qubit-qubit coupling via indirect RKKY interaction mediated by Luttinger liquid of electrons, and we assume spin- $\frac{1}{2}$ qubits.

It has been commonly accepted⁸ that the ground state excitations of a 1D interacting electron gas within the Luttinger liquid model can be described by the following Hamiltonian:

$$H_0 = \sum_{i=c,s} \frac{v_i}{4\pi} \int dx [g_i (\partial_x \phi_i)^2 + g_i^{-1} (\partial_x \theta_i)^2]. \quad (1)$$

The phase fields, $\phi_{c(s)}(x)$ and $\theta_{c(s)}(x)$, with subscript indices describing charge and spin degrees of freedom, respectively, obey commutation relations $[\partial_x \phi_{c(s)}(x), \theta_{c(s)}(x')] = 2\pi i \delta(x-x')$. Consequently, θ and $\partial_x \phi$ can be viewed as canonical variables. Here and in what follows we set $\hbar=1$ and $k_B=1$.

The Hamiltonian has a simple additive structure as a result of spin-charge separation in 1D systems, with the charge and spin density waves of the liquid having, generally speaking, different velocities, $v_c=v_F/g_c$ and $v_s=v_F/g_s$, respectively, where v_F is the Fermi velocity. The constant $g_c>0$ accounts for the electron-electron interaction and is related to the parameters of the Hubbard model⁹ as follows: $g_c \simeq (1 + U/2E_F)^{-1/2}$, where $E_F=v_F k_F/2$ is the Fermi energy, k_F is the Fermi momentum, and U is the effective interaction between the electrons, $U \sim e^2/a$, where a is the short distance cutoff, $a \sim k_F^{-1}$. Also, we assume rotational symmetry, SU(2), in the spin space,⁹⁻¹¹ which implies that $g_s=1$.

The localized magnetic moments (spins) are coupled to conduction electrons via the contact interaction,

$$H_{\text{int}} = \sum_j J_j \mathbf{s}(x_j) \cdot \mathbf{S}_j. \quad (2)$$

Here $j=1,2$ labels impurity spins \mathbf{S}_j positioned at x_j , J_j are the exchange coupling constants, and $\mathbf{s}(x)$ is the local electron spin density. The spin density can be explicitly expressed in terms of the Luttinger phase fields, see Refs. 12 and 13,

$$s_z = \frac{\partial_x \theta_s}{2\pi} + \frac{\sigma_z}{\pi a} \cos(2k_F x + \theta_c) \cos \theta_s, \quad (3)$$

$$s_{\pm} = \frac{e^{\pm i \phi_s}}{\pi a} [\pm i \sigma_y \cos \theta_s + \sigma_x \cos(2k_F x + \theta_c)], \quad (4)$$

where $\sigma_{x,y,z}$ are the Pauli matrixes. The Luttinger liquid description of the problem is generally valid in the “hydrodynamic limit” of spin separations $x=|x_1-x_2| \gg a$.

Our goal is to obtain an effective description of the dynamics of the system of two spins, with electronic degrees of freedom integrated out. We first consider the equilibrium partition function of the system, defined as $Z = \text{Tr}[\exp(-\beta H)]$, where $\beta=1/T$ is the inverse temperature and $H=H_0+H_{\text{int}}$. The partition function, Z , can be expressed in terms of the spin- and temperature-dependent effective action \mathcal{S}_{eff} , see Ref. 14, $Z = \int \mathcal{D}\mathbf{S}_1 \mathcal{D}\mathbf{S}_2 \exp[-\mathcal{S}_{\text{eff}}(\mathbf{S}_1, \mathbf{S}_2)]$. As usual, the evaluation of the effective action can only be carried out perturbatively in the couplings J_j of the spins to the electrons. The leading nonvanishing contribution is generated by

$(1/2)\int_0^\beta d\tau_1 d\tau_2 \langle \mathcal{T} H_{\text{int}}(\tau_1) H_{\text{int}}(\tau_2) \rangle_{H_0}$, where \mathcal{T} stands for Matsubara time ordering,¹⁵ and the equilibrium averaging is taken with respect to the noninteracting Hamiltonian H_0 . The perturbative approach is, generally speaking, invalid at sufficiently large Matsubara time scales, i.e., low temperatures, as evident from the perturbative RG analysis carried out in Refs. 11 and 12. At sufficiently low temperatures, spin dynamics results in a nontrivial strong coupling fixed point, i.e., Kondo effect. For quantum information processing, we are interested in qubits that retain their localized spin- $\frac{1}{2}$ identity. Therefore, we limit our consideration to temperatures $T > T_{\text{Kondo}}$.

The resulting Matsubara action is $\mathcal{S}_{\text{eff}} = \mathcal{S}_{\text{Berry}} + \mathcal{S}_{\text{self}}$. Here $\mathcal{S}_{\text{Berry}}$ is the Berry action term for free spins.¹⁶ The self-action for the spins is given by

$$\mathcal{S}_{\text{self}} = \frac{J^2}{2\beta} \sum_{i\omega_n} [\chi(\omega_n, 0) \mathbf{S}_1(i\omega_n) \cdot \mathbf{S}_1(-i\omega_n) + \chi(\omega_n, x) \mathbf{S}_1(i\omega_n) \cdot \mathbf{S}_2(-i\omega_n) + (\mathbf{S}_1 \leftrightarrow \mathbf{S}_2)]. \quad (5)$$

In (5), $\chi(\omega_n, x)$ is the spin-spin correlation function of the electron gas (in the imaginary time representation), and $\omega_n = 2\pi n/\beta$ are the Matsubara frequencies. For simplicity, in (5) and in the following we assume that $J_1 = J_2 = J$. The correlation function $\chi(\omega_n, x)$, which generally speaking is a tensor, $\langle s_i(-i\omega_n, x) s_j(i\omega_n, 0) \rangle$, in the SU(2) symmetric case reduces to a scalar function, $\chi(\omega_n, x) = \langle s_z(-i\omega_n, x) s_z(i\omega_n, 0) \rangle$. It can be evaluated by using (1) and (4). Assuming that the electron gas is dense to satisfy $E_F \gg \beta^{-1}$, one obtains (with $g \equiv g_c$),

$$\chi(\omega_n, x) = -\frac{|\omega_n|}{4\pi v_F^2} \exp\left(-\frac{|\omega_n x|}{v_F}\right) + \frac{\cos(2k_F x)}{(2\pi)^{g+1} a^{1-g}} \int \frac{d\tau \exp(-i\omega_n \tau)}{(x^2 + v_F^2 \tau^2)^{1/2} (x^2 + v_F^2 \tau^2 / g^2)^{g/2}}. \quad (6)$$

The correlation function in (6) contains two contributions, one due to forward scattering, peaked at wave vector $q=0$, and another due to backwards scattering, peaked at $q = \pm 2k_F$. As expected, the forward scattering contribution is g -independent.

In the perturbative regime considered, the dynamics of the spins is slow and controlled by small parameter J^2/v_F^2 . Therefore, we can use the small-frequency asymptotic form of the correlation function $\chi(\omega_n, x)$ for $|\omega_n x|/v_F \ll 1$,

$$\chi(\omega_n, x) \approx \frac{C_1(g) \cos(2k_F x)}{v_F a^{1-g} x^g} - \frac{1}{4\pi v_F^2} \left[|\omega_n| + \frac{C_2(g) \cos(2k_F x) |\omega_n|^g}{v_F^{g-1} a^{1-g}} \right], \quad (7)$$

where $C_1 = (2\pi)^{-g-1} g^g \int dz (1+z^2)^{-1/2} (g^2+z^2)^{-g/2}$ and $C_2 = 4(2\pi)^{-g} g^{g-1} \Gamma(1-g) \sin[(\pi/2)(1-g)]$. The first term in (7) corresponds to interaction between the spins. The interaction is oscillatory and decays as a power law x^{-g} . This result is consistent with Ref. 13.

The second, ω_n dependent term in (7) corresponds to relaxation of the spins. To demonstrate this property, we perform a transition to the real time dynamics of the spins according to analytical continuation rule, see Ref. 17. We introduce a standard Keldysh contour with forward and return branches time-ordered and anti-time-ordered, respectively. In the Keldysh representation, the effective action, (5), reads

$$\frac{J^2}{2} \int \frac{d\omega}{2\pi} [\bar{\mathbf{S}}_1^T(\omega) \hat{\chi}(\omega, x) \bar{\mathbf{S}}_1(-\omega) + \bar{\mathbf{S}}_1^T(\omega) \hat{\chi}(\omega, 0) \bar{\mathbf{S}}_2(-\omega) + (\bar{\mathbf{S}}_1 \leftrightarrow \bar{\mathbf{S}}_2)]. \quad (8)$$

Here $\bar{\mathbf{S}}_{i=1,2}$ are two-element column vectors, with spin-vector operators as elements, such that their transposes are $\bar{\mathbf{S}}_i^T = (\mathbf{S}_{ic}, \mathbf{S}_{iq})$. The ‘‘classical,’’ \mathbf{S}_{ic} , and ‘‘quantum,’’ \mathbf{S}_{iq} , components of the spin \mathbf{S}_i are the following combinations of spin operators on the forward (f) and return (r) branches of the Keldysh contour, $\mathbf{S}_{ic} = (\mathbf{S}_{if} + \mathbf{S}_{ir})/2$ and $\mathbf{S}_{iq} = \mathbf{S}_{if} - \mathbf{S}_{ir}$. The response (correlation) function $\hat{\chi}(\omega, x)$ is then a 2×2 matrix, which can be expressed in terms of the retarded and advanced response functions, χ^R and χ^A ,

$$\hat{\chi} = \begin{pmatrix} 0 & \chi^A \\ \chi^R & \chi^K \end{pmatrix}. \quad (9)$$

The retarded and advanced response functions are related to the Matsubara response function via the analytic continuation $i\omega \rightarrow \omega \pm i\delta$. In thermal equilibrium χ^K can be expressed in terms of the retarded and advanced components by using the fluctuation-dissipation theorem, $\chi^K = \coth(\beta\omega/2)(\chi^R - \chi^A)$.

Let us consider first the noninteracting case, $g=1$, and later we will extend the results to $g \neq 1$. For noninteracting electrons, the response function corresponds to an Ohmic heat bath, with $C_1(1) = 1/(4\pi)$ and $C_2(1) = 1$ in (7). Upon Fourier transform, (8) yields several terms. Those containing products $\mathbf{S}_{ic} \cdot \mathbf{S}_{jq}$ represent interaction between spins, while the $\dot{\mathbf{S}}_{ic} \cdot \mathbf{S}_{jq}$ and $\mathbf{S}_{iq} \cdot \dot{\mathbf{S}}_{jq}$ terms are responsible for energy dissipation and pure dephasing (decoherence), respectively. The dissipative (time-derivative) terms are small and can be neglected here. Indeed, since $\dot{\mathbf{S}}_i \sim O(J^2/v_F^2)$, the $\dot{\mathbf{S}}$ -dependent terms in the action in (8) are of order J^4/v_F^4 . The resulting dynamics of the spins is governed by the action $\mathcal{S}_{\text{self}}^K = \mathcal{S}_{\text{int}}^K + \mathcal{S}_{\text{dec}}^K$, where

$$\mathcal{S}_{\text{int}}^K = \mathcal{J}_{\text{eff}} \int dt [\mathbf{S}_{1c}(t) \cdot \mathbf{S}_{2q}(t) + \mathbf{S}_{1q}(t) \cdot \mathbf{S}_{2c}(t)], \quad (10)$$

$$\mathcal{S}_{\text{dec}}^K = \frac{i\gamma}{2\pi} \int dt_1 dt_2 \int_0^\infty d\omega \omega \cos[\omega(t_2 - t_1)] \coth\left(\frac{\beta\omega}{2}\right) \times \{2\mathbf{S}_{1q}(t_1) \cdot \mathbf{S}_{1q}(t_2) - [1 + \cos(2k_F x)] \times \mathbf{S}_{1q}(t_1) \cdot \mathbf{S}_{2q}(t_2) + [\mathbf{S}_{1q} \leftrightarrow \mathbf{S}_{2q}]\}, \quad (11)$$

with $\mathcal{J}_{\text{eff}} = J^2 \cos(2k_F x)/(4\pi v_F x)$ and $\gamma = J^2/(2\pi v_F^2)$.

Equation (10) corresponds to coherent interaction of the spins according to the scalar $\mathbf{S}_1 \cdot \mathbf{S}_2$ coupling. Equation (11) represents quantum noise resulting from thermal fluctuations

of the Luttinger liquid. The noise, generally speaking, is colored. Moreover, the noises experienced by the two spins are correlated.

The action in (11) can be simplified further if we recall that the dynamics of the spins is slow, i.e., $\mathbf{S}_{iq}(t_2) = \mathbf{S}_{iq}(t_1) + O(J^2/v_F^2)$. Then, the self-action can be replaced by an instantaneous action, by setting $\mathbf{S}_{iq}(t_1) = \mathbf{S}_{iq}(t_2)$ in (11), which now corresponds to the white noise source. The full action of the spin system can now be written as

$$\mathcal{S}^K = \mathcal{S}_{\text{Berry}}^K - \int dt H^K(\mathbf{S}_1, \mathbf{S}_2), \quad (12)$$

where the Berry action plays the role of a velocity \times momentum term in the Lagrangian-Hamiltonian transformation.¹⁶ The ‘‘Keldysh Hamiltonian’’ in (12) is

$$\begin{aligned} H^K = & -\mathcal{J}_{\text{eff}}(\mathbf{S}_{1f} \cdot \mathbf{S}_{2f} - \mathbf{S}_{1r} \cdot \mathbf{S}_{2r}) \\ & - 4i\gamma T[(\sin \phi \mathbf{S}_{1q} - \cos \phi \mathbf{S}_{2q})^2 + (\mathbf{S}_{1q} \leftrightarrow \mathbf{S}_{2q})], \end{aligned} \quad (13)$$

where $0 \leq \phi \leq \pi/2$ is defined by $\sin(2\phi) = \cos^2(k_F x)$.

The density matrix of the two-spin system evolves according to the ‘‘Schrödinger equation’’ $\dot{\rho} = -i\mathcal{T}_K(H^K\rho)$, where Keldysh time-ordering implies that the ‘‘forward’’ operators, with subscript f , are positioned to the left of the density matrix, while the ‘‘return’’ operators, labelled by r , are to the right of the density matrix,

$$\begin{aligned} \dot{\rho} = & i\mathcal{J}_{\text{eff}}[\mathbf{S}_1 \cdot \mathbf{S}_2, \rho] - 4\gamma T \sum_{\alpha=x,y,z} ([\sin \phi S_1^\alpha - \cos \phi S_2^\alpha, [\sin \phi S_1^\alpha \\ & - \cos \phi S_2^\alpha, \rho]] + (\mathbf{S}_1 \leftrightarrow \mathbf{S}_2)). \end{aligned} \quad (14)$$

This expression contains both the mediated interaction and dephasing due to the electron environment.

It is instructive to illustrate the dynamics of the two-spin system, described by (14), for a particular initial state of the system, $|\uparrow\downarrow\rangle$. Without the quantum noise term, proportional to γT in (14), the $\mathbf{S}_1 \cdot \mathbf{S}_2$ interaction would split the singlet and triplet spin states. As a result, the system would oscillate between the $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$ states with frequency determined by the singlet-triplet energy gap. The effects of the noise include damping of these oscillations, as illustrated in Fig. 1. Furthermore, for $t > 0$, the system subject to the noise will no longer remain in a pure quantum state. The departure of the resulting mixed state from a pure state can be measured by deviation of $\text{Tr}[\rho^2(t)]$ from the pure-state value of 1, as illustrated in Fig. 2. We point out that effective evolution equations that involve only commutators, linear in the density matrix, on the right-hand side (RHS), typically fail to reproduce thermal equilibrium at large times. Instead, as seen in Figs. 1 and 2—note the asymptotic values—the fully random mixed state is obtained. Therefore, the present approximation should not be used beyond the relaxation time defined by (14), namely, it only applies for $t < 1/[T(J^2/v_F^2)]$, and the theory is therefore applicable in the regime of interest for quantum computing applications, for short and intermediate times, because both factors in the denominator, T

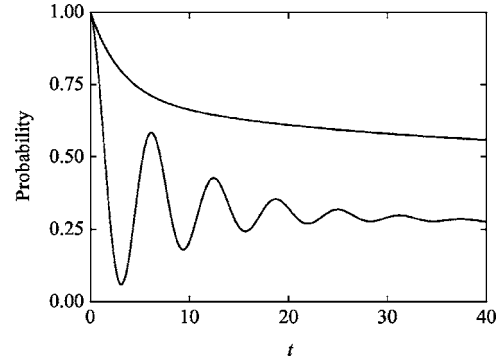


FIG. 1. Lower curve, the probability, $\rho_{|\uparrow\downarrow\rangle,|\uparrow\downarrow\rangle}$, to find the two-spin system in the $|\uparrow\downarrow\rangle$ state as a function of time, for a convenient set of parameter values, $\mathcal{J}_{\text{eff}}=1$, $\gamma T=0.0125$, $k_F x=\pi$. Upper curve, the sum of the probabilities to find the two-spin system in the states $|\uparrow\downarrow\rangle$ or $|\downarrow\uparrow\rangle$, given by $\rho_{|\uparrow\downarrow\rangle,|\uparrow\downarrow\rangle} + \rho_{|\downarrow\uparrow\rangle,|\downarrow\uparrow\rangle}$. The large-time limiting values of these probabilities are $\frac{1}{4}$ and $\frac{1}{2}$, respectively.

and J^2/v_F^2 , are small. Similar results have been obtained in Ref. 18, studying mediated interaction and decoherence due to noninteracting electron gas.

Finally, we extend our results to the interacting case, $g \neq 1$. Modification of the interaction term, (10), is straightforward and comes from the first term on the RHS of (7). Equation (10) applies with

$$\mathcal{J}_{\text{eff}}(g) = C_1(g) J^2 \frac{\cos(2k_F x)}{v_F a^{1-g} x^g}. \quad (15)$$

Thus, indirect interaction between spins becomes longer range as a result of electron-electron interaction.

Modification of the decoherence term, (11), is less obvious. Consider for simplicity a single spin situation, when the first term on the RHS of (7) can be omitted, and in the second term we can set $x=0$. The $|\omega_n|$ term then in the large

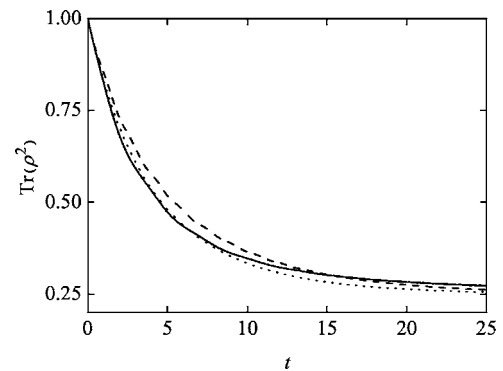


FIG. 2. Solid line, the quantity $\text{Tr}(\rho^2)$, with the initial value of 1 corresponding to a pure state, as a function of time for the same parameter values as in Fig. 1, with positive $\mathcal{J}_{\text{eff}}=1$. Dashed line, parameter values illustrating the case of negative $\mathcal{J}_{\text{eff}}=-2$, with $\gamma T=0.0125$, $k_F x=\pi/2$. Dotted line, the deviation from a pure state (the quantum noise effect) is present even when the leading-order induced interaction is $\mathcal{J}_{\text{eff}}=0$, with $\gamma T=0.0125$, $k_F x=\pi/4$. The curves for $\mathcal{J}_{\text{eff}} \neq 0$ have weak oscillations superimposed on the decay. The large-time limiting values are $\frac{1}{4}$.

parentheses in (7) produces the same, g -independent, contribution to the decoherence rate as in the noninteracting case. The $|\omega_n|^g$ term in the large parentheses requires careful consideration. Indeed, we are primarily interested in the repulsive, $U > 0$, Hubbard-model interaction, i.e., $0 < g < 1$. The two-spin contribution then yields a divergence: after setting $\mathbf{S}_{iq}(t_1) = \mathbf{S}_{iq}(t_2)$ in (11) and integrating over $t_1 - t_2$, the frequency integral $\int d\omega \omega^g \delta(\omega) \coth(\beta\omega/2)$ is divergent for $g < 1$. The origin of this divergence is actually not in the instantaneous assumption but instead it can be traced back to too liberal a use of the large- β approximation in our evaluation of the response function in (6), and specifically, extending the limits of the τ integration from $-\infty$ to ∞ , while we should have integrated from $-\beta$ to β . The response function in (6) can be readily evaluated for $\omega_n \rightarrow 0$ (and $x=0$) for finite β as well. After a straightforward repetition of steps leading to Eq. (6) one obtains the dephasing rate for a paramagnetic spin imbedded in Luttinger liquid,

$$\tau_{\text{dec}}^{-1} = 4\gamma \left(T + \frac{C_3(g)T^g}{v_F^{g-1}a^{1-g}} \right), \quad (16)$$

where $C_3(g) = 2g^g \sin[\pi(1-g)] / [(2\pi)^g(1-g)]$. The second term on the RHS of Eq. (16) (due to backscattering) represents modification of the Korringa law for spin relaxation in the interacting 1D electron gas. Thus, electron-electron repulsion leads to the reduction of spin relaxation rate.

In summary, our main result, (14), is the first theoretically derived dynamical equation that incorporates both the coherent RKKY-type induced interaction and the effects of quantum noise due to interacting electrons in the 1D conduction channel. The approximations and assumptions involved, limit our results to temperatures in the range $T_K < T \ll T_F$. Furthermore, to have the noise term small, the temperature should be actually in $T_K < T < v_F/x$, where x is the qubit (spin) separation; restoring the constants earlier set to 1, the upper bound here is $\hbar v_F/k_B x$. The relative strength of the interaction vs noise terms can be also controlled by positioning of the qubits, owing to the oscillatory dependence on x , typical for RKKY-coupled systems.

The authors acknowledge helpful discussions with D. F. James, I. Martin, and A. Shnirman. The work was supported by US DOE, and by the NSF, Grant No. DMR-0121146.

-
- ¹D. Loss and D. P. DiVincenzo, Phys. Rev. A **57**, 120 (1998); A. Imamoglu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. **83**, 4204 (1999); R. Vrijen, E. Yablonovitch, K. Wang, H. W. Jiang, A. Balandin, V. Roychowdhury, T. Mor and D. DiVincenzo, Phys. Rev. A **62**, 012306 (2000).
- ²V. Privman, I. D. Vagner, and G. Kventsel, Phys. Lett. A **239**, 141 (1998); D. Mozyrsky, V. Privman, and M. L. Glasser, Phys. Rev. Lett. **86**, 5112 (2001); D. Mozyrsky, V. Privman, and I. D. Vagner, Phys. Rev. B **63**, 085313 (2001); C. Piermarocchi, P. Chen, L. J. Sham, and D. G. Steel, Phys. Rev. Lett. **89**, 167402 (2002); G. Ramon, Y. Lyanda-Geller, T. L. Reinecke, and L. J. Sham, Phys. Rev. B **71**, 121305(R) (2005); M. G. Vavilov and L. I. Glazman, Phys. Rev. Lett. **94**, 086805 (2005).
- ³V. Privman, D. Mozyrsky, and I. D. Vagner, Comput. Phys. Commun. **146**, 331 (2002).
- ⁴C. Kittel, *Quantum Theory of Solids* (Wiley, New York, 1987).
- ⁵N. J. Craig, J. M. Taylor, E. A. Lester, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Science **304**, 565 (2004).
- ⁶M. Xiao, I. Martin, E. Yablonovitch, and H. W. Jiang, Nature **430**, 435 (2004); J. M. Elzerman, R. Hanson, L. H. Willems van Beveren, B. Witkamp, L. M. K. Vandersypen, and L. P. Kouwenhoven, *ibid.* **430**, 431 (2004); M. R. Sakr, E. Yablonovitch, E. T. Croke, and H. W. Jiang, cond-mat/0504046 (unpublished).
- ⁷M. Hilke, D. C. Tsui, L. N. Pfeiffer, and K. W. West, J. Phys. Soc. Jpn. **72**, 92 (2003); A. M. Chang, L. N. Pfeiffer, and K. W. West, Phys. Rev. Lett. **77**, 2538 (1996).
- ⁸F. D. M. Haldane, J. Phys. C **14**, 2585 (1981); J. Sólyom, Adv. Phys. **28**, 201 (1970).
- ⁹N. Nagaosa, *Quantum Field Theory in Strongly Correlated Electronic Systems* (Springer-Verlag, Berlin, 1999).
- ¹⁰H. J. Schulz, in *Mesoscopic Quantum Physics*, edited by Akkermans *et al.*, Les Houches Session LXI (Elsevier, Amsterdam, 1995).
- ¹¹C. L. Kane and M. P. A. Fisher, Phys. Rev. B **46**, 15233 (1992).
- ¹²D. H. Lee and J. Toner, Phys. Rev. Lett. **69**, 3378 (1992).
- ¹³R. Egger and H. Schoeller, Phys. Rev. B **54**, 16337 (1996).
- ¹⁴N. Nagaosa, *Quantum Field Theory in Condensed Matter Physics* (Springer-Verlag, Berlin, 1999).
- ¹⁵G. D. Mahan, *Many-Particle Physics* (Kluwer, New York, 2000).
- ¹⁶A. Shnirman, Z. Nussinov, J.-X. Zhu, A. V. Balatsky, and Y. Makhlin, Low Temp. Phys. **30**, 629 (2004) [Fiz. Nizk. Temp. **30**, 834 (2004)].
- ¹⁷A. Kamenev, cond-mat/0412296 (unpublished).
- ¹⁸Y. Rikitake and H. Imamura, Phys. Rev. B **72**, 033308 (2005).