Clarkson Calculus Readiness Test 18P -- **Solutions**

1. Find an equation for the line through the point (1,-7) with slope -2.

   Solution: The point-slope form of the equation is $y-(-7)=-2(x-1)$. This can be transformed into $y=-2x-5$.

2. Expand $(2x+3)^2$.

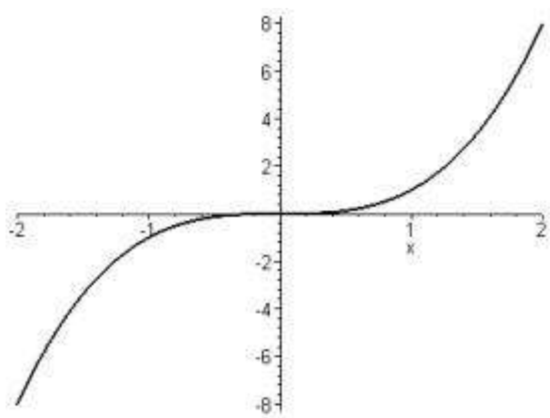
   Solution: $(2x+3)(2x+3)=4x^2+6x+6x+9=4x^2+12x+9\ (by\ FOIL)$

3. Rewrite using fractional exponents and simplify $3a\sqrt{12a}-\sqrt{(4a)^3}$.

   Solution: $\sqrt{12a}=2*3^{1/2}a^{1/2}$, $\sqrt{(4a)^3}=4^{3/2}a^{3/2}=8a^{3/2}$. Combine to find $3a\sqrt{12a}-\sqrt{(4a)^3}$ =3*$2*3^{1/2}a^{1/2}-8a^{3/2}=2(3^{3/2}-4)a^{3/2}$. Usually we would write $3\sqrt{3}$ for $3^{3/2}$.

4. Sketch the graph of $y=x^3$ for $-2<x<2$. Label the scales on the axes.

   Solution:



   This is a computer-generated graph. You should be able to *sketch* a reasonably accurate version without a calculator.

5. At what value of x does the line $2x+3y=7$ intersect the line $y=1$?

   Solution: If y=1, the value of x on the line satisfies 2x+3=7, so x=2.

6. Factor the polynomial x^3+x^2-x-1, given that $x=1$ is a root.

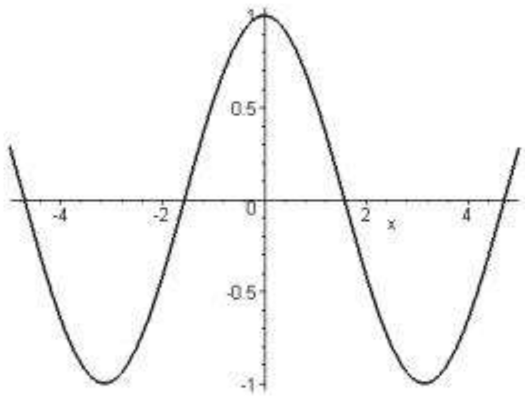
   Solution: Since x=1 is a root (check: substitutuing 1 for x gives the value 0 for the polynomial), the polynomial can be divided by (x-1). The result of the division is x^2+2x+1, which is $(x+1)^2$. The factors are x-1 and $(x+1)^2$.

7. Solve $128 + 16t - 16t^2 = 0$.

Solution: It is easier if you divide through be -16 first, so that the equation to solve is $t^2 - t - 8 = 0$. Then solving using the quadratic formula to get $t = \frac{1 \pm \sqrt{(-1)^2 - 4(-8)}}{2} = \frac{1 \pm \sqrt{33}}{2}$

8. Sketch the graph of $y = \cos(x)$ for $-5 < x < 5$. Label the scales on the axes.

Solution:



This is a computer-generated graph. You should be able to *sketch* a reasonably accurate version without a calculator, by plotting the values of cosines at 0, 30, 45, 60, 90 degrees, etc. (multiples of pi/6 and pi/4). You should also know the shape, intercepts, and height of the graph.

9. If $\cos(A) = -0.6$, and $0 < A < \pi$, find the numerical values of $\sin(A)$ and $\tan(A)$.

Solution: Since $cos^2(A) + sin^2(A) = 1$ for any angle A, we have $(-.6)^2 + sin^2(A) = 1$. Thus $\sin(A) = .8$ or -.8. The angle A is between 0 and pi, where the sine is positive , so sin (A) = +.8. Then tan(A)=sin(A)/cos(A)=.8/(-.6)=-4/3.

10. The short leg of a 30-60-90triangle measures 7 cm. How long are the long leg and hypotenuse?

Solution: In a 30-60-90 triangle, the sides have lengths in the ratio $1\!:\!2\!:\!\sqrt{3}$, so in this triangle the ratios are $7\!:\!14\!:\!7\sqrt{3}$ for the short leg, hypotenuse, and long leg.

11. Solve $2x + 7 > 3$ for x.

Solution: Subtract 7 from each member of the inequality to get 2x>-4. Next divide both members by 2 to get x>-2.

12. Find the surface area of a cylindrical can with diameter 3 inches and height 6 inches.

Solution: The cylindrical surface has area $2\pi rh$ and each end has area πr^2, so the total surface area is $2\pi rh + 2\pi r^2 = 22.5\pi$. Remember that the diameter is 3, so the radius is 3/2.

13. For $f(x) = \frac{x}{x+1}$, find $\frac{f(x+h)-f(h)}{h}$ and simplify.

Solution: $\frac{x+h}{x+h+1} - \frac{x}{x+1} = \frac{(x+h)(x+1)}{(x+h+1)(x+1)} - \frac{x(x+h+1)}{(x+1)(x+h+1)} = \frac{x^2+x+hx+h-(x^2+hx+x)}{(x+h+1)(x+1)} = \frac{h}{(x+h+1)(x+1)}$ So the solution is $\frac{1}{(x+h+1)(x+1)}$

14. Find the numerical value of sin $(7\pi/6)$.

Solution: sin(7π/6) = sin(210) = -1/2. You should know sines and cosines of special angles (multiples of pi/4 and pi/6) by heart.

15. Complete the square: $x^2 + 4x + 1$.

Solution: $x^2 + 4x + 1 = (x+2)^2 - 3$

16. Solve for all values of *x*: $\frac{1}{x} - \frac{1}{x+1} = \frac{1}{6}$.

Solution: $\frac{1}{x} - \frac{1}{x+1} = \frac{x+1-x}{x(x+1)}$ so the equation to solve is $\frac{1}{x(x+1)} = \frac{1}{6}\ or\ x(x+1) = 6$. Put this in standard form, $x^2 = x - 6 = 0$ and factor or use the quadratic formula to get x=-2 or x—3.

17. A poster with dimensions *L* by *W* is to be made with these specifications: margins of one inch on each side; printed area inside the margins 180 sq. in. Find a formula that gives the length in terms of the width.

Solution: The printed area has dimensions L-2 by W-2, so (L-2)(W-2)=180. Solve this equation first for L-2 then for L to get L = 180/(W-2)-2.

18. If the length of the poster is to be twice the width, find the dimensions of the poster.

Solution: Since L = 2W, the area specification is (2W-2)(W-2)=180, or (W-1)(W-2)=90. Expand the left and then put in standard form for a quadratic equation: $W^2 - 3W - 88 = 0$. Use the quadratic formula (or factor the polynomial) to find the roots +11 and -8 for W. Since W is a width, only the positive root is valid. The dimensions are 11 by 22.